# Mathematics (Project work)

CLASS - X

Q.1. If $-5$ is a root of the quadratic equation $2x^2+px-15=0$ and the quadratic equation $p(x^2+x)+K=0$ has equal roots, find the value of K. Ans. [$K = 7/4$]

Q.2. If the equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$ has equal roots, prove that $c^2=a^2(1+m^2)$.

Q3. Find the value(s) of p for which the quadratic equation $(2p+1)x^2-(7p+2)x+(7p-3)=0$ has equal roots. Also find these roots.

[$4, -\frac{4}{7}$; when $p=4$: $5/3, 5/3$; when $p=-4/7$: $7, 7$]

Q4. Solve the equation $5x^2-3x-4=0$ and give your answer correct to 3 significant figures. [$1.24, -0.643$]

Q.5. Solve the equation $2x-\frac{1}{x}=7$ and give your answer correct to two decimal places. [$3.64, -0.14$]

Q.6. Find a and b if $\begin{bmatrix} a-b & b-4 \\ b+4 & a-2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}=\begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$, $a=2$, $b=3$.

Q.7. Given matrix $A=\begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$ and $B=\begin{bmatrix} 4 \\ 5 \end{bmatrix}$. If ~~AB~~ $AX=B$

(i) Write the order of matrix X. (ii) Find the matrix X.

Q 8. If $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$, find $A^2 - 2AB + B^2$ and $(A-B)^2$.

Q.9. In an A.P. fourth and sixth terms are 8 and 14 respectively find the : (a) first term (b) common difference (c) sum of the first 20 terms.

[12, 7, 2]
or
3, 7, 11.

Q.10. If m times the mth term of an A.P. is equal to n times the nth term show that $(m+n)$th term of the A.P is 0.

Q.11. If the sum of first 7 terms of an A.P. is 49 and that of the 17 terms is 289, find the sum of first n terms.

Q.12. The Sum of first 16 terms of an A.P is 112 and the sum of its next 14 terms is 518. Find the A.P.

Q.13. If the first term of an A.P is 2 and the sum of the first five terms is equal to one fourth of the next five terms, then.
(i) show that $t_{20} = 112$ (ii) find the sum of first 30 terms.

14. How many terms of the A.P. 17, 15, 13, ....... must be added to get the sum 72? Explain the double answer.

15. Determine the number of terms n in the G.P. $T_1, T_2, T_3 \ldots T_n$ if $T_1 = 3$, $T_n = 96$ and $S_n = 189$.

Q 16. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Find the sum of n terms of the G.P.

$\left[\frac{16}{7}(2^n - 1)\right.$

Q.17. Determine the number of terms of a G.P., ~~if $T_1 = 3$~~ $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ must be taken to make the sum equal to 728?

[6.]

Q.18. The sum of some terms of a G.P is 315 whose first term and the common ratio are 5 and 2 respectively. Find the last term and the number of terms.

$\left[\begin{array}{l} n = 6 \\ l = 160. \end{array}\right.$

Q.19. If $\frac{a}{b} = \frac{c}{d}$, show that $a+b : c+d = \sqrt{a^2+b^2} : \sqrt{c^2+d^2}$

Q 20. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, then prove that $\frac{ax - by}{(a+b)(x-y)} = 1.$

Q21. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, show that

(i) $\frac{x^3}{a^3} - \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{xyz}{abc}$

(ii) $\left(\frac{a^2x^2 + b^2y^2 + c^2z^2}{a^3x + b^3y + c^3z}\right)^{3/2} = \sqrt{\frac{xyz}{abc}}$

(iii) $\frac{x^2 + y^2 + z^2}{a^2 + b^2 + c^2} = \left(\frac{px + qy + rz}{pa + qb + rc}\right)^2$

Q.22. If a, b, c, d are in continued Proportion, prove that:

(i) $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$

(ii) $\sqrt{ab} - \sqrt{bc} + \sqrt{cd} = \sqrt{(a-b+c)(b-c+d)}$

(iii) $(b-c)^2 + (c-a)^2 + (d-b)^2 = (a-d)^2$

Q.23. If $x = \dfrac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} - \sqrt{a^2-b^2}}$, show that $b^2x^2 - 2a^2x + b^2 = 0$

Q.24. If $\dfrac{a^3+3ab^2}{3a^2b+b^3} = \dfrac{x^3+3xy^2}{3x^2y+y^3}$, show that $\dfrac{x}{a} = \dfrac{y}{b}$.

Q.25. If $y = \dfrac{(p+1)^{1/3} + (p-1)^{1/3}}{(p+1)^{1/3} - (p-1)^{1/3}}$, find the value of $y^3 - 3py^2 + 3y - p$

Q.26. If $y = \dfrac{\sqrt{a+3b} + \sqrt{a-3b}}{\sqrt{a+3b} - \sqrt{a-3b}}$, show that $3by^2 - 2ay + 3b = 0$

Q.27. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x-1$ and $x+1$, the remainders are respectively 5 and 19. Determine the remainder when $f(x)$ is divided by $(x-2)$. [$a=5$, $b=8$, Rem. = 10.

Q.28. If $R_1$ and $R_2$ are the remainders when polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x+1$ and $x-2$ respectively. If $2R_1 + R_2 = 6$, find the value of $a$. [$a=2$]

Q.29. Using factor theorem factorise the polynomial

$f(x) = 2x^3 + 3x^2 - 9x - 10$. Completely.

Q.30 Given that $(x+1)$ and $(x-2)$ are factors of the polynomial $f(x) = x^3 + ax^2 - bx - 6$, find the values of a and b. With these values of a and b, factorize the polynomial completely.

[$a = 2, b = 5$]

Q.31. The angle of elevation of the top of a tower 30 m. high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the tower.

[$10\sqrt{3}$ m, 10 m.

Q.32. The angle of elevation of the top of a vertical tower from a point on the ground is 60° from another point 10 m. vertically above the first, its angle of elevation is 45°. Find the height of the tower.

[$5(\sqrt{3}+3)$ m.]

Q.33. The angle of elevation of a cloud from a point 60 m. above a lake is 30° and the angle of depression of the reflection of cloud in the lake is 60°. Find the height of the cloud. [120 m.]

Q. 34. From a window 15 m. high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are $30^\circ$ and $45^\circ$ respectively show that the height of the opposite house is 24.66 m. ($\sqrt{3} = 1.732$).

Q. 35. A tree 12 m. high is broken by the wind in such a way that its top touches the ground and makes an angle $60^\circ$ with the ground. At what height from the bottom the tree is broken by the wind? [5.569 m.

Q. 36. Prove that:

(i) $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

(ii) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

Q. 37. Prove that:

(i) $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1+\sin\theta}{\cos\theta}$ ; (ii) $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \dfrac{1+\cos\theta}{\sin\theta}$

Q. 38. Prove that: $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin^2 A}{\sin A - \cos A} = \sin A + \cos A$.

Q. 39. Prove that: $\dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A} = 1 + \sec A \operatorname{cosec} A$.

Q. 40. Prove that: $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1 + \sec\theta \operatorname{cosec}\theta)^2$